INVESTIGATION OF OBLIQUE COLLISIONS OF METAL PLATES IN AN ELASTIC FORMULATION

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In connection with the investigation of different processes of explosive treatment of materials [1], interest has recently grown in the investigation of collisions between solids, particularly metals, at velocities on the order of several hundred meters per second. New physical phenomena, for example, wave formation on the contact surfaces, are observed in the case of oblique collisions when the contact point moves along the contact surfaces at a definite velocity. Hydrodynamic models, which are surveyed in [1], are used for the theoretical investigations. The metals are hence considered ideal fluids, similar to problems about cumulation [2]. The use of hydrodynamic computational schemes permits computation of the magnitude of the pressure in the neighborhood of the contact point; the computed values obtained are used in various problems of explosive welding [1]. At the same time, the investigations of some new phenomena disclosed the inadequacy of using a hydrodynamic model; in particular, there are foundations to assume that the magnitude of the tengential stresses and the time of their existence play a substantial role in processes associated with the change in metal properties near the collision zone.

An attempt is made in this paper to investigate the other extreme case as compared to the hydrodynamic model, to consider the problem of an oblique collision between plates within the framework of linear elasticity theory. The single attempt to use this approach in explosive-welding problems which is known to us is the paper [3], in which the solution of the problem of collisions in an elastic formulation is used to clarify the wave-formation process during explosive welding. The thickness of the colliding plates was hence considered infinite, and the velocity of the contact point was assumed greater than the velocity of longitudinal wave propagation in the material. The elastic problem will be considered in this paper in a substantially more general formulation.

1. FORMULATION OF THE PROBLEM

Let two elastic plates move oppositely, where $\vec{v}_1$ is the velocity of the upper plate, and $\vec{v}_2$ is the velocity of the lower plate, and their directions are perpendicular to the plate surfaces. The plates are joined, as shown in Fig. 1, because of the collision. To simplify the problem, let us consider the plates to consist of the same material and to be of the same thickness h, while the collision is symmetrical. The plates can be considered flat and to have constant velocities equal to $\vec{v}_1$ and $\vec{v}_2$, respectively, far to the right of the collision zone (see Fig. 1). It should be noted that the existence of elastic waves moving more rapidly than the contact zone and not being damped at great distances is possible in certain plate-collision modes. Such waves, which originate during load motion along the surface of an elastic finite-thickness plate, have been studied in [4]. However, in this case the plate surfaces far from the contact zone can be represented as vibrating around certain "principal surfaces" which move at the constant velocities $\vec{v}_1$ and $\vec{v}_2$. Hence, unless stipulated otherwise, the velocity of namely such "principal surfaces" will be understood to be the plate velocity at infinity.

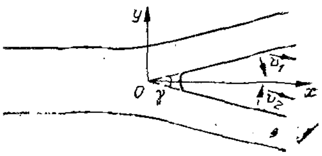

Fig. 1

Let us continue the inner surfaces of the plates just as is shown in Fig. 1. Their intersection in the plane of the sketch yields the point 0, which we call the contact point. The angle γ formed because of the intersections of the planes will be called the collision angle. The magnitude of the contact-point velocity V, when the hurling velocities are directed along the normals to the plates and are equal in magnitude, will be defined by the following expression:

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$$V = \frac{|\vec{v}_1|}{\sin\gamma/2} = \frac{|\vec{v}_2|}{\sin\gamma/2} = \frac{v_0}{\sin\gamma/2}. \tag{1.1}$$

It is convenient to analyze the collision process in a coordinate system coupled to the contact point. The origin will be placed at the contact point, and the x axis will be directed along the bisectrix of the collision angle. To eliminate edge effects from the problem, we shall consider the plates sufficiently long and the collision process itself to be stationary.

Let us examine the case most characteristic for the majority of explosive welding modes when the two plates collide at a small angle [1]. Under the assumption of smallness of the collision angles, the problem about collision can be solved without taking account any change in shape of the plate boundary. Let us consider the upper boundary to be the line $y=h$ and the lower to be the line $y=-h$, and the collision angle to be represented by a slit along the x axis from the origin to infinity. Such an idealization of the problem under consideration in the ideal fluid scheme has been examined in [1].

The boundary conditions and the conditions at infinity are written as follows:

$$\sigma_{22}=\sigma_{12}=0, \quad (y=h, \ -\infty<x<\infty), \tag{1.2}$$
$$\sigma_{22}=\sigma_{12}=0, \quad (y=0, \ 0<x<\infty),$$
$$\sigma_{22}=\sigma_{12}=0, \quad (y=-h, \ -\infty<x<\infty),$$
$$u=0, \ v=0, \ (x=-\infty), \tag{1.3}$$
$$u=0, \ v=-v_0\cos\gamma/2, \ (x=\infty, \ y>0),$$
$$u=0, \ v=v_0\cos\gamma/2, \ (x=\infty, \ y<0).$$

Here σ_{ik} are the stress tensor components, and u, v are the components of the displacement velocity vector along the x and y axes, respectively. The existence of an integrable singularity for both σ_{ik} and also u and v is assumed at the contact point.

In order to obtain the expressions for the components of the stress tensor σ_{ik} and the displacement velocity vector u, v, let us use scalar φ and vector ψ potentials. These potentials satisfy the equations

$$\left(1-\frac{\rho V^2}{K+\frac{4}{3}\mu}\right)\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=0, \tag{1.4}$$
$$\left(1-\frac{\rho V^2}{\mu}\right)\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=0,$$

where K and μ are the multilateral compression and the shear moduli, and ρ is the density of the material.

It follows from the form of (1.4) that the picture of the collision depends essentially on the magnitude of the contact point velocity V. If this latter exceeds the longitudinal wave velocity $c_1=\sqrt{(K+{}^4/_3\,\mu)/\rho}$, a situation occurs which it is natural to call supersonic flow. In this case the system (1.4) is hyperbolic and admits of seeking the solution by the method of characteristics. The other case of intersonic motion originates when the velocity V is less than the logitudinal wave velocity c_1 by greater than the transverse wave velocity $c_2=\sqrt{\mu/\rho}$. Under this condition, the first equation in (1.4) is elliptic and the second is hyperbolic. Finally, the last case, subsonic motion, is realized if the velocity V of the contact point is less than the transverse wave velocity c_2. The system (1.4) is elliptic in this collision mode.

2. COLLISION OF PLATES IN THE SUBSONIC MODE

Let us seek the solution of the problem posed above in the subsonic collision case by the Fourier method in combination with the Wiener–Hopf method. Let $f_1(k, y)$, $f_2(k, y)$ denote the Fourier transforms of the potentials φ and ψ, respectively, and let σ^0_{22} be the stress acting on the interface between the materials. Moreover, let us introduce the notation

$$p(k)=\frac{V}{2\mu}\int_{-\infty}^{0}\sigma_{22}^0(x)\,e^{ikx}dx; \tag{2.1}$$

$$b(k)=2\int_0^\infty v(x,0)\,e^{ikx}dx; \tag{2.2}$$

$$\lambda_1=\sqrt{1-\frac{\rho V^2}{\mu}}; \quad \lambda_2=\sqrt{1-\frac{\rho V^2}{K+\frac{4}{3}\mu}}; \quad \delta=1-\frac{\rho V^2}{2\mu}.$$

Solving (1.4) with the boundary conditions (1.2), we obtain the following expressions for the Fourier transforms of the potentials:

$$f_1(k, y) = -\frac{p(k)}{Vk^2 I(k)}[\delta^3 \operatorname{sh}(k\lambda_1 h)\operatorname{sh}(k\lambda_2(h-y)) - \tag{2.3}$$

$$-\lambda_1\lambda_2\delta \operatorname{ch}(k\lambda_1 h)\operatorname{ch}(k\lambda_2(h-y)) + \lambda_1\lambda_2\delta \operatorname{ch}(k\lambda_2 y)];$$

$$f_2(k, y) = \frac{ip(k)}{\lambda_1 Vk^2 I(k)}[-\lambda_1^2\lambda_2^2 \operatorname{sh}(k\lambda_2 h)\operatorname{ch}(k\lambda_1(h-y)) +$$

$$+\delta^2\lambda_1\lambda_2 \operatorname{ch}(k\lambda_2 h)\operatorname{sh}(k\lambda_1(h-y)) + \delta^2\lambda_1\lambda_2 \operatorname{sh}(k\lambda_1 y)];$$

$$I(k) = \left(\delta^4 + \lambda_1^2\lambda_2^2\right)\operatorname{sh}(k\lambda_1 h)\operatorname{sh}(k\lambda_2 h) + 2\delta^2\lambda_1\lambda_2 - 2\delta^2\lambda_1\lambda_2 \operatorname{ch}(k\lambda_1 h)\operatorname{ch}(k\lambda_2 h).$$

Taking account of the continuity of the stresses and velocities on the interface between the materials, we obtain the Wiener–Hopf equation

$$b(k) = \frac{2i\lambda_2(\delta-1)p(k)}{I(k)}[\lambda_1\lambda_2 \operatorname{sh}(k\lambda_2 h)\operatorname{ch}(k\lambda_1 h) - \delta^2 \operatorname{ch}(k\lambda_2 h)\operatorname{sh}(k\lambda_1 h)]. \tag{2.4}$$

Now, let us turn to an investigation of the analytic properties of the functions p(k) and b(k), defined by the integrals (2.1) and (2.2) in order to provide a possibility for the solution of (2.4) for the two unknown functions p(k) and b(k) by the Wiener–Hopf method.

It is known from the theory of nonstationary elastic waves that a load moving along the surface of an elastic plate can radiate an undamped elastic wave with a phase velocity equal to the load velocity. Hence, if the load velocity is less than the velocity of the Rayleigh waves in the material, then the group velocity of the radiated elastic wave will be greater than the phase velocity, and conversely in the opposite case. In the stationary formulation of this problem this means that in the mode when the velocity V of the contact point is less than the velocity c_R of the Rayleigh waves, the stresses in a strip far in front of the contact point are bounded and oscillate, while the stresses damp out exponentially far behind the contact point as they recede from the origin, where the exponent of the exponential depends on the plate thickness. In the mode for which $V > c_R$, the situation becomes the reverse: behind the contact point there is an undamped elastic wave, while the stresses damp out exponentially ahead of the contact point.

It follows from the above that the integral (2.1) determines the function p(k) in a half-plane $\mathrm{Im}(k) < \eta$ where $\eta > 0$ in case $V < c_R$, and depends on the velocity of stress damping at infinity. The integral (2.3), in turn, determines the function b(k) in the half-plane $\mathrm{Im}(k) \geq 0$ with the exception of several points on the real axis. Therefore, the Wiener–Hopf equation (2.4) is valid in some strip of the plane k. Despite the fact that this strip does not completely enclose the real axis of the complex k plane, (2.4) can be solved by the Wiener–Hopf method, since singular points are successfully excluded from the desired strip of regularity, in the case of meromorphic functions, by redefining the functions. The foundation for the possibility of solving the equation in the collision mode when $c_R < V < c_2$ is carried out in an analogous manner.

Now, let z_m, z_j', k_n denote, respectively, the roots of the equations

$$A(k) = \lambda_1\lambda_2 \operatorname{sh}(k\lambda_2 h)\cdot\operatorname{ch}(k\lambda_1 h) - \delta^2 \operatorname{ch}(k\lambda_2 h)\operatorname{sh}(k\lambda_1 h) = 0; \tag{2.5}$$

$$B(k) = \lambda_1\lambda_2 \operatorname{ch}(k\lambda_2 h)\operatorname{sh}(k\lambda_1 h) - \delta^2 \operatorname{sh}(k\lambda_2 h)\cdot\operatorname{ch}(k\lambda_1 h) = 0; \tag{2.6}$$

$$I(k) = 4\cdot A\left(\frac{k}{2}\right)\cdot B\left(\frac{k}{2}\right) = 0, \tag{2.7}$$

in the upper half-plane of the complex variable k which does not include the real axis. Let z_0, z_0', k_0 denote the roots of the corresponding equations (2.5), (2.6), (2.7) on the real axis. Then taking account the results mentioned above about the singular points on the real axis, we obtain the following relationships from (2.4):

$$p(k)\frac{\prod\limits_{m=1}^{\infty}\left(1-\frac{k}{z_m}\right)e^{\frac{k}{z_m}}}{\prod\limits_{n=1}^{\infty}\left(1-\frac{k}{k_n}\right)e^{\frac{k}{k_n}}} = -b(k)\frac{\prod\limits_{n=1}^{\infty}\left(1+\frac{k}{k_n}\right)e^{-\frac{k}{k_n}}}{\prod\limits_{m=1}^{\infty}\left(1+\frac{k}{z_m}\right)e^{-\frac{k}{z_m}}}\frac{ih\left(\lambda_1^2-\delta^2\right)k}{2(\delta-1)}\left(1-\frac{k^2}{k_0^2}\right) = p_1(k), \quad \text{if} \quad \frac{\lambda_1\lambda_2}{\delta^2} > 1; \tag{2.8}$$

$$p(k)\frac{\left(1-\frac{k^2}{z_0^2}\right)\prod\limits_{m=1}^{\infty}\left(1-\frac{k}{z_m}\right)e^{\frac{k}{z_m}}}{\left(1-\frac{k^2}{k_0^2}\right)\prod\limits_{n=1}^{\infty}\left(1-\frac{k}{k_n}\right)e^{\frac{k}{k_n}}} = -b(k)\frac{\prod\limits_{n=1}^{\infty}\left(1+\frac{k}{k_n}\right)e^{-\frac{k}{k_n}}}{\prod\limits_{m=1}^{\infty}\left(1+\frac{k}{z_m}\right)e^{-\frac{k}{z_m}}}\frac{ih\left(\lambda_1^2-\delta^2\right)k}{2(\delta-1)} = p_1(k), \quad \text{if} \quad \frac{\lambda_1\lambda_2}{\delta^2} < 1.$$

Here $p_1(k)$ is an unknown entire function, and the symbol $\prod\limits_{n=1}^{\infty}$ denotes the infinite product evaluated over all the appropriate roots z_m, k_n.

Because of the assumption about the existence of an integrable singularity at the contact point, the functions p(k) and b(k) should decrease to zero in their domain of definition as $|k| \to \infty$. At the same time the ratio between the infinite products in (2.8) will not grow more rapidly than an exponential for large $|k|$. Considering the asymptotic of the product of the function $p(k)b(-k)$ in their domain of definition as $|k| \to \infty$, it can be obtained that

$$p(k) \to (i-1)\frac{C_1 k_0}{\sqrt{k}}\sqrt{\frac{\lambda_1\lambda_2-\delta^2}{2h\lambda_2\left(\delta^2-\lambda_1^2\right)}}; \tag{2.9}$$

$$b(k) \to (1+i)\frac{2(\delta-1)C_1k_0}{h\left(\lambda_1^2-\delta^2\right)\sqrt{k}}\sqrt{\frac{h\lambda_2\left(\delta^2-\lambda_1^2\right)}{2\left(\lambda_1\lambda_2-\delta^2\right)}},$$

as follows from (2.8). Then, using the Liouville theorem, it can be proved that the function $p_1(k)$ is

$$p_1(k) = (C_1k + C_2)\,e^{C_3k} \text{ for } \lambda_1\lambda_2 > \delta^2; \qquad p_1(k) = C_4e^{C_5k} \text{ for } \lambda_1\lambda_2 < \delta^2. \tag{2.10}$$

In order to determine the constants C_3 and C_5, the asymptotic of the infinite products in (2.8) must be investigated and then values must be selected for C_3 and C_5 such that the asymptotic of p(k) would have no oscillating terms and would decrease as $|k| \to \infty$. The values of the constants C_2 and C_4 can be determined from the condition that the plate velocities far in front of the contact points would eaual the hurling velocities:

$$C_2 = C_4 = -\frac{v_0\cos(\gamma/2)\,h\left(\lambda_1^2-\delta^2\right)}{\delta-1}. \tag{2.11}$$

Let us note that if the integrals (2.1) and (2.2) have an asymptotic form of the type (2.9), then the stresses σ_{22}^0 near the contact point are determined on the continuation of the slit by the relationship

$$\sigma_{22}{}^0 = \frac{2i\mu C_1k_0}{V\sqrt{-x}}\sqrt{\frac{\lambda_1\lambda_2-\delta^2}{\pi h\lambda_2\left(\delta^2-\lambda_1^2\right)}}, \tag{2.12}$$

and the elastic displacements of the edges of the slit near the origin will equal

$$Y(x) = -\frac{2i(\delta-1)C_1k_0}{hV\left(\lambda_1^2-\delta^2\right)}\sqrt{\frac{h\lambda_2\left(\delta^2-\lambda_1^2\right)}{\pi\left(\lambda_1\lambda_2-\delta^2\right)}}\sqrt{x}.$$

Let stresses of equal magnitude and opposite direction acting on a finite segment of length Δl be applied to the slit edges at some time

$$\sigma_{22} = \frac{2i\mu C_1k_0}{V\sqrt{\pi(\Delta l-x)}}\sqrt{\frac{\lambda_1\lambda_2-\delta^2}{h\lambda_2\left(\delta^2-\lambda_1^2\right)}}.$$

As a result of such a load the slit edges merge and are connected in such a way that the contact point is provisionally displaced a distance Δl from its previous location. The applied loads, hence, perform nonzero work equal to

$$A' = 2\cdot\frac{1}{2}\int_0^{\Delta l}\frac{4(\delta-1)C_1^2k_0^2\mu}{\pi hV^2\left(\lambda_1^2-\delta^2\right)}\sqrt{\frac{x}{\Delta l-x}}\,dx = -\frac{\rho C_1^2k_0^2\Delta l}{h\left(\lambda_1^2-\delta^2\right)}. \tag{2.13}$$

It has therefore been shown that the solution of (2.4) which contains the nonzero constant C_1 corresponds to the solution of the problem about the collision of elastic plates with energy absorption occuring at the contact point. A small plastic zone adjacent to the contact point, or processes associated with surface friction, etc., can be such an "energy absorber" in the actual explosive-welding process. Additional investigations are needed to determine the magnitude of the energy U which is lost during the welding of unit lengths of plates. After having determined the quantity U, the constant C_1 can be evaluated by using the relationship

$$U = -\frac{\rho C_1^2k_0^2}{h\left(\lambda_1^2-\delta^2\right)}. \tag{2.14}$$

Such a method of finding the constant was used in [7] to determine the stress intensity factor near the vertex of a stationary crack in an infinite material.

It is quite difficult to use (2.8) containing the infinite products; however, the difficulties are diminished considerably if the ratio λ_1/λ_2 is considered a rational number n/l, where n and l are arbitrary positive odd numbers. Then the roots of (2.5)-(2.7) are periodic, and the infinite products are successfully represented by gamma functions, namely: if $\eta_1, \eta_2, \ldots, \eta_{l+n-4}/2$, $[(-\pi i/2)+\beta]$ are the roots of the equation $A(\eta n/\lambda_1 h) = 0$ in the strip $0 > \operatorname{Im}\eta \geq -(\pi/2)$, and β is a real number, then the expressions for p(k) will be

$$p(k) = \frac{\rho v_0 V^2 h\beta \cos(\gamma/2)}{\pi^{3/2}\mu} \sqrt{\frac{\lambda_1\lambda_2 - \delta^2}{l\left(\delta^2 - \lambda_1^2\right)}}\, 2^{-\frac{ik(\lambda_1+\lambda_2)}{\pi}} \cdot \Phi(k) \frac{\Gamma\left(\frac{1}{2} - \frac{ik\lambda_1 h}{n\pi}\right)}{\Gamma\left(1 - \frac{ik\lambda_1 h}{n\pi}\right)} \prod_{j=1}^{\frac{l+n-4}{2}} \frac{\Gamma\left(\frac{i\eta_j}{\pi} - \frac{ik\lambda_1 h}{n\pi}\right)\Gamma\left(1 - \frac{i\eta_j}{\pi} - \frac{ik\lambda_1 h}{n\pi}\right)}{\Gamma\left(\frac{2i\eta_j}{\pi} - \frac{ik\lambda_1 h}{n\pi}\right)\Gamma\left(1 - \frac{2i\eta_j}{\pi} - \frac{ik\lambda_1 h}{n\pi}\right)}; \quad (2.15)$$

if $\lambda_1\lambda_2 < \delta^2$, then

$$\Phi(k) = \left(\frac{2C_1\mu k}{\rho v_0 V^2 h \cos(\gamma/2)} - 1\right) e^{C_3 k} \frac{\Gamma\left(\frac{1}{2} - \frac{i\beta}{\pi} - \frac{ik\lambda_1 h}{n\pi}\right)}{\Gamma\left(1 - \frac{2i\beta}{\pi} - \frac{ik\lambda_1 h}{n\pi}\right)} \frac{\Gamma\left(\frac{1}{2} + \frac{i\beta}{\pi} - \frac{ik\lambda_1 h}{n\pi}\right)}{\Gamma\left(1 + \frac{2i\beta}{\pi} - \frac{ik\lambda_1 h}{n\pi}\right)};$$

if $\lambda_1\lambda_2 < \delta^2$, then

$$\Phi(k) = \frac{\pi}{4i\beta} e^{C_5 k} \frac{\Gamma\left(-\frac{i\beta}{\pi} - \frac{ik\lambda_1 h}{n\pi}\right)\Gamma\left(\frac{i\beta}{\pi} - \frac{ik\lambda_1 h}{n\pi}\right)}{\Gamma\left(-\frac{2i\beta}{\pi} - \frac{ik\lambda_1 h}{n\pi}\right)\Gamma\left(\frac{2i\beta}{\pi} - \frac{ik\lambda_1 h}{n\pi}\right)}.$$

Investigating the asymptotic behavior of the function (2.1) for large $|k|$, it can be noted that the function $p(k)$ has a power-law type asymptotic only if

$$C_3 = C_5 = \frac{i(\lambda_1 + \lambda_2)\ln 2}{\pi}.$$

Therefore, we not only reduced the formula defining $p(k)$ to a more convenient form, but also evaluated the values of the constants C_3 and C_5.

3. INVESTIGATION OF SOLUTIONS IN THE SUBSONIC COLLISION CASE

The recovery of the potentials φ, ψ from their Fourier transforms (2.3) can be done by numerical inversion of the Fourier transforms. However, a qualitative picture of the flow and an analysis of the other stresses in the neighborhood of the contact point can be obtained directly from (2.3) and (2.1).

One of the most essential qualitative distinctions of plate collisions in an elastic formulation from the hydrodynamic model presented in [1] is that the stresses and displacements in the elastic problem are oscillatory along the x axis at infinity. This is associated with a resonance phenomenon in which the load moving at a velocity V along the surface of an elastic strip of thickness h generates a natural elastic wave with phase velocity equal to the load velocity in this plate. The presence of elastic standing waves results in the appearance of tensile stresses at some distance from the contact point. Rupture of the connection being formed can occur during explosive welding when the magnitude of the rupturing stresses in the butt is sufficiently high. This phenomenon is apparently related to the existence of an "upper bound" to the welding domain [9].

Now, let us turn to an investigation of the elastic stress field in the neighborhood of the contact point. Using the reasoning that there is just one point in the problem where the stresses and displacement velocities can have a singularity, the nature of this singularity can be investigated by using just high-frequency harmonics of the Fourier transformation. Consequently, we obtain the following expressions in polar coordinates:

$$\sigma_{11} = -\frac{2\mu a}{V} \operatorname{Re}\left\{\frac{i\lambda_1\lambda_2}{\sqrt{\cos\varphi + i\lambda_1 \sin\varphi}} - \frac{i\delta\left(2 - \delta + \lambda_2^2\right)}{\sqrt{\cos\varphi + i\lambda_2\sin\varphi}}\right\};$$

$$\sigma_{22} = \frac{2\mu a}{V} \operatorname{Re}\left\{\frac{i\lambda_1\lambda_2}{\sqrt{\cos\varphi + i\lambda_1\sin\varphi}} - \frac{i\delta^2}{\sqrt{\cos\varphi + i\lambda_2\sin\varphi}}\right\};$$

$$\sigma_{12} = -\frac{2\mu a}{V} \operatorname{Im}\left\{\frac{i\delta\lambda_2}{\sqrt{\cos\varphi + i\lambda_1\sin\varphi}} - \frac{i\delta\lambda_2}{\sqrt{\cos\varphi + i\lambda_2\sin\varphi}}\right\};$$

$$p = -\rho V a\left(1 - \frac{4}{3}\frac{c_2^2}{c_1^2}\right) \operatorname{Re}\frac{i}{\sqrt{\cos\varphi + i\lambda_2\sin\varphi}};$$

$$y(r) = \frac{\lambda_2(1-\delta) r a}{V};$$

$$a = \sqrt{\frac{U}{\rho h}} \frac{\sqrt{h}|\lambda_1\lambda_2 - \delta_2|}{(\delta^2 - \lambda_1\lambda_2)\sqrt{\pi\lambda_2 r}}, \quad \text{if} \quad \lambda_1\lambda_2 > \delta^2;$$

$$a = \frac{v_0\cos(\gamma/2)\sqrt{h}\,|\lambda_1\lambda_2 - \delta^2|}{2(\delta^2 - \lambda_1\lambda_2)\sqrt{\pi\lambda_2 r}}, \quad \text{if} \quad \lambda_1\lambda_2 < \delta^2$$

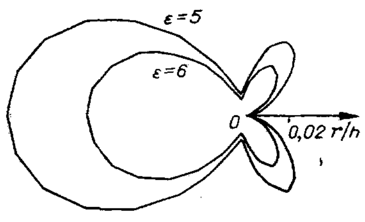

Fig. 2

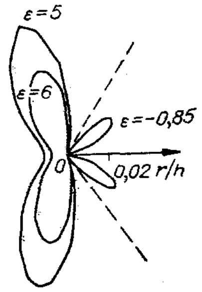

Fig. 3

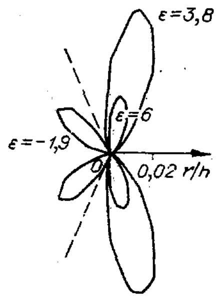

Fig. 4

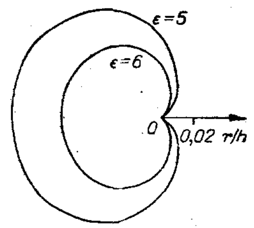

Fig. 5

for the stress tensor components σ_{ij}, the pressure $p = -(\sigma_{ii}/3)$, and the displacements y(r) of the slit edges in the neighborhood of the contact point. Curves of the constant dimensionless stresses σ_{11}, σ_{22}, σ_{12}, and the pressure $\left(\varepsilon = \dfrac{\sigma_{ij}}{\sqrt{\dfrac{\rho l^2 U}{h}}}\right)$ are presented in Figs. 2-5, respectively.

Let us note that the displacement y(r) is negative for a contact point velocity less than the velocity of the Rayleigh wave in the material. This means that the slit edges switch about. Therefore, the contact point cannot move at a constant velocity less than the Rayleigh wave velocity. The velocity of the contact point will apparently perform some fluctuations around a mean value equal to V.

A quantitative comparison between the elastic model and its hydrodynamic analog [1] can be obtained by comparing the pressure intensity factors

$$\frac{p_{\text{hydr}}}{p_{\text{ela}}} = \frac{\sqrt{\dfrac{\lambda_1\lambda_2}{\delta^2} - 1}}{1 - \dfrac{4}{3}\dfrac{c_2^2}{c_1^2}}.$$

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